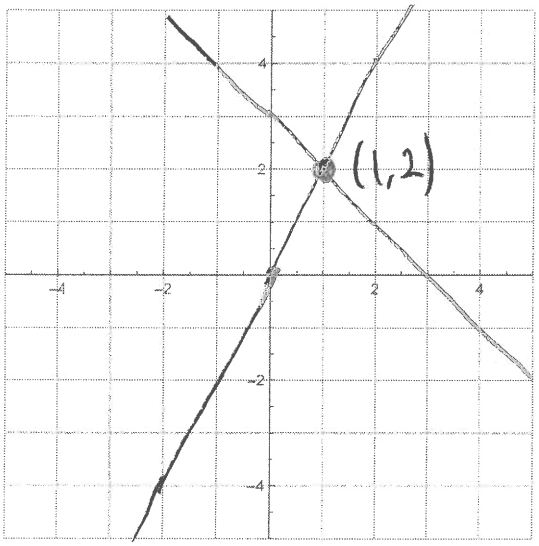
Finite Math - Spring 2017

Lecture Notes - 3/20/2017

Problem 1. *Solve the system of equations using the graphing method*

$$\begin{aligned} x + y &= 3 \\ 2x - y &= 0 \end{aligned}$$



$x+y=3$

$x=0 \to y=3$ (0,3)

$y=0 \to x=3$ (3,0)

$2x-y=0$

$x=0 \to y=0$ (0,0)

$x=1 \to y=2$ (1,2)

Check (1,2)

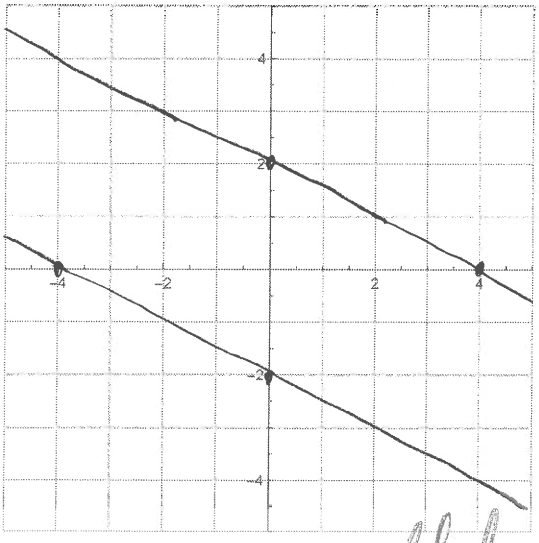
$1+2=3$ ✓

$2(1)-2=2-2=0$ ✓

Sol'n: (1,2)

Problem 2. *Solve the system of equations using the graphing method*

$$\begin{aligned} x + 2y &= 4 \\ 2x + 4y &= -8 \end{aligned}$$



$x+2y=4$

$x=0 \to y=2$ (0,2)

$y=0 \to x=4$ (4,0)

$2x+4y=-8$

$x=0 \to y=-2$ (0,-2)

$y=0 \to x=-4$ (-4,0)

parallel ⇒ no solution

Problem 3. *Solve the system of equations*

$$\begin{aligned} 3x - 2y &= 12 \quad (1) \\ 7x + 2y &= 8 \quad (2) \end{aligned}$$

(1)+(2): $10x = 20 \Rightarrow x = 2$

Plug into (1): $3(2) - 2y = 12$

$6 - 2y = 12$

$-2y = 6$

$y = -3$

Sol: $(2, -3)$

Problem 4. *Solve the system of equations*

$$\begin{aligned} -6x + 10y &= -30 \quad (1) \\ 3x - 5y &= 15 \quad (2) \end{aligned}$$

$$\begin{array}{rl} (1) & -6x + 10y = -30 \\ +2(2) & 6x - 10y = 30 \\ \hline & 0 + 0 = 0 \end{array}$$

inf. sol.

(2): $3x - 5y = 15$

$3x = 5y + 15$

$x = \frac{1}{3}(5y + 15)$

Let $y = t$, then

$x = \frac{1}{3}(5t + 15)$

$y = t$

Problem 5. *Animals in an experiment are to be kept under a strict diet. Each animal should receive 20 grams of protein and 6 grams of fat. The laboratory technician is able to purchase two food mixes: Mix A has 10% protein and 6% fat; mix B has 20% protein and 2% fat. How many grams of each mix should be used to obtain the right diet for one animal?*

	Mix A	Mix B	Total
protein	.1	.2	20
fat	.06	.02	6

Let x = # grams mix A
y = # grams mix B

$.1x + .2y = 20$ ①
$.06x + .02y = 6$ ②

① $.1x + .2y = 20$
$-10$② $-.6x - .2y = -60$
$-.5x = -40$

$\Rightarrow x = 80$

Plug into ①:
$.1(80) + .2y = 20$
$8 + .2y = 20$
$.2y = 12$
$y = 60$

Use 80g of mix A and 60g of mix B.

Problem 6. *Solve the system of equations using Gauss-Jordan elimination*

$$\begin{aligned} x + 3y &= 1 \\ 3x - 2y &= 14 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 3 & -2 & 14 \end{array}\right] \xrightarrow{R_2 - 3R_1 \to R_2} \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -11 & 11 \end{array}\right] \xrightarrow{-\frac{1}{11}R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{R_1 - 3R_2 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array}\right]$$

$x = 4$
$y = -1$

Problem 7. *Solve the system of equations using Gauss-Jordan elimination*

$$\begin{aligned} 2x &- 4y = -2 \\ -3x &+ 6y = 3 \end{aligned}$$

$$\begin{bmatrix} 2 & -4 & | & -2 \\ -3 & 6 & | & 3 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & -2 & | & -1 \\ -3 & 6 & | & 3 \end{bmatrix} \xrightarrow{R_2 + 3R_1 \to R_2} \begin{bmatrix} 1 & -2 & | & -1 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$x - 2y = -1$

$\Rightarrow x = 2y - 1$

$x = 2t - 1$

$y = t$

Problem 8. *Solve the system of equations using Gauss-Jordan elimination*

$$\begin{aligned} 3x &+ 8y - z = -18 \\ 2x &+ y + 5z = 8 \\ 2x &+ 4y + 2z = -4 \end{aligned}$$

$$\begin{bmatrix} 3 & 8 & -1 & | & -18 \\ 2 & 1 & 5 & | & 8 \\ 2 & 4 & 2 & | & -4 \end{bmatrix} \begin{matrix} R_1 - R_2 \to R_1 \\ R_3 - R_2 \to R_3 \end{matrix} \begin{bmatrix} 1 & 7 & -6 & | & -26 \\ 2 & 1 & 5 & | & 8 \\ 0 & 3 & -3 & | & -12 \end{bmatrix} \xrightarrow{R_2 - 2R_1 \to R_2} \begin{bmatrix} 1 & 7 & -6 & | & -26 \\ 0 & -13 & 17 & | & 60 \\ 0 & 3 & -3 & | & -12 \end{bmatrix}$$

$$-R_2 - 4R_3 \to R_2 \quad \begin{matrix} 0 & 13 & -17 & -60 \\ 0 & -12 & 12 & 48 \end{matrix} \quad \begin{bmatrix} 1 & 7 & -6 & | & -26 \\ 0 & 1 & -5 & | & -12 \\ 0 & 3 & -3 & | & -12 \end{bmatrix} \begin{matrix} R_1 - 7R_2 \to R_1 \\ R_3 - 3R_2 \to R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 29 & | & 58 \\ 0 & 1 & -5 & | & -12 \\ 0 & 0 & 12 & | & 24 \end{bmatrix}$$

$$\frac{1}{12}R_3 \to R_3 \begin{bmatrix} 1 & 0 & 29 & | & 58 \\ 0 & 1 & -5 & | & -12 \\ 0 & 0 & 1 & | & 2 \end{bmatrix} \begin{matrix} R_1 - 29R_3 \to R_1 \\ R_2 + 5R_3 \to R_2 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 1 & | & 2 \end{bmatrix}$$

$x = 0, y = -2, z = 2$

Problem 9. *Solve the system of equations using Gauss-Jordan elimination*

$$\begin{aligned} 2x &- y - 3z = 8 \\ x &- 2y \quad\quad = 7 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & -3 & 8 \\ 1 & -2 & 0 & 7 \end{array}\right] \xrightarrow{R_1 - R_2 \to R_1} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 1 & -2 & 0 & 7 \end{array}\right] \xrightarrow{R_2 - R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & -3 & 3 & 6 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{3}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & -1 & -2 \end{array}\right] \xrightarrow{R_1 - R_2 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

$x - 2z = 3$

$y - z = -2$

Let $z = t$

$$\begin{aligned} x &= 2t + 3 \\ y &= t - 2 \\ z &= t \end{aligned}$$

Problem 10. *Solve the system of equations using Gauss-Jordan elimination*

$$\begin{aligned} 4x &- y + 2z = 3 \\ -4x &+ y - 3z = -10 \\ 8x &- 2y + 9z = -1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 4 & -1 & 2 & 3 \\ -4 & 1 & -3 & -10 \\ 8 & -2 & 9 & -1 \end{array}\right] \xrightarrow{\frac{1}{4}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & -1/4 & 1/2 & 3/4 \\ -4 & 1 & -3 & -10 \\ 8 & -2 & 9 & -1 \end{array}\right]$$

$$\begin{matrix} R_2 + 4R_1 \to R_2 \\ R_3 - 8R_1 \to R_3 \end{matrix} \left[\begin{array}{ccc|c} 1 & -1/4 & 1/2 & 3/4 \\ 0 & 0 & -1 & -7 \\ 0 & 0 & 5 & 5 \end{array}\right] \xrightarrow{-R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -1/4 & 1/2 & 3/4 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 5 & 5 \end{array}\right]$$

$$\xrightarrow{R_3 - 5R_2 \to R_3} \left[\begin{array}{ccc|c} 1 & -1/4 & 1/2 & 3/4 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & -30 \end{array}\right]$$

NO solution

Problem 11. *A chemical manufacturer wants to lease a fleet of* 24 *railroad tank cars with a combined carry capacity of* 520,000 *gallons. Tank cars with three different carrying capacities are available:* 8,000 *gallons,* 16,000 *gallons, and* 24,000 *gallons. How many of each type of tank car should be leased?*

x = # of 8000 gal cars

y = # of 16000 gal cars

z = # of 24000 gal cars

total # of cars: $x+y+z=24$

total capacity: $8000x+16000y+24000z=520000$

$\div 8000$ → $x+2y+3z=65$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 1 & 2 & 3 & 65 \end{array}\right] \xrightarrow{R_2-R_1\to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 0 & 1 & 2 & 41 \end{array}\right] \xrightarrow{R_1-R_2\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -17 \\ 0 & 1 & 2 & 41 \end{array}\right]$$

$x-z=-17 \to x=z-17$ Let $z=t$

$y+2z=41 \to y=41-2z$

$x=t-17,\ y=41-2t,\ z=t$

$t\geq 17$ $t\leq 20$ $t\geq 0$ $t=17,18,19,20$

The company should rent $(t-17)$ 8,000 gal cars, $(41-2t)$ 16,000 gal cars, and t 24,000 gal cars where $t=17,18,19,20$